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Parameter Estimation for Markov-Modulated Infinite-Server Queues

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The Markov-modulated infinite-server queue is a queueing system with infinitely many servers, where the arrivals follow a Markov-modulated Poisson process (MMPP), i.e. a Poisson process with rate modulating between several values. The modulation is driven by an underlying and unobserved continuous time Markov chain $\{X_t\}_{t\geq 0}$. The inhomogeneous rate of the Poisson process, $\lambda(t)$, stochastically alternates between d different rates, $\lambda_1, \ldots, \lambda_d$, in such a way that $\lambda(t) = \lambda_i$ if $X_t = i$, $i = 1, \ldots, d$.

We are interested in estimating the parameters of the arrival process for this queueing system based on observations of the queue length at discrete times only. We assume exponentially distributed service times with rate μ, where μ is time-independent and known. Estimation of the parameters of the arrival process has not yet been studied for this particular queueing system. Two types of missing data are intrinsic to the model, which complicates the estimation problem. First, the underlying continuous time Markov chain in the Markov-modulated arrival process is not observed. Second, the queue length is only observed at a finite number of discrete time points. As a result, it is not possible to distinguish the number of arrivals and the number of departures between two consecutive observations.

In this talk we show how we derive an explicit algorithm to find maximum likelihood estimates of the parameters of the arrival process, making use of the EM algorithm. Our approach extends the one used in Okamura et al. (2009), where the parameters of an MMPP are estimated based on observations of the process at discrete times. However, in contrast to our setting, Okamura et al. (2009) do not consider departures and therefore do not deal with the second type of missing data. We illustrate the accuracy of the proposed estimation algorithm with a simulation study.

Reference: Okamura H., Dohi T., Trivedi K.S. (2009). Markovian Arrival Process Parameter Estimation With Group Data. IEEE/ACM Transactions on Networking. Vol. 17, No. 4, pp. 1326–1339